ISSN 2348-3156 (Print)
International Journal of Social Science and Humanities Research ISSN 2348-3164 (online)
Vol. 8, Issue 1, pp: (493-498), Month: January - March 2020, Available at: www.researchpublish.com

APPLICATION AND CRITISM OF MEAN VARIANCE THEORY

Otieno Wesley Okoth

PhD Finance Candidate, Jomo Kenyatta University of Agriculture and Technology and Lecturer at RAF International University

Abstract: **The fundamental concept behind MVT is that the assets in an investment portfolio should not be selected individually, each on its own merits. Rather, it is important to consider how each asset changes in price relative to how every other asset in the portfolio changes in price. Investing is a tradeoff between risk and expected return. In general, assets with higher expected returns are riskier. The stocks in an efficient portfolio are chosen depending on the investor's risk tolerance, an efficient portfolio is said to be having a combination of at least two stocks above the minimum variance portfolio. For a given amount of risk, MVT describes how to select a portfolio with the highest possible expected return. Or, for a given expected return, MVT explains how to select a portfolio with the lowest possible risk.**

Keywords: **Mean Variance Theory (MVT), Portfolio, Return, Risk.**

1. INTRODUCTION

Harry Markowitz introduced MVT in a 1952 article and a 1959 book. Markowitz classifies it simply as "Portfolio Theory".

Definition: is a theory of finance that attempts to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return, by carefully choosing the proportions of various assets.

MVT is a mathematical formulation of the concept of diversification in investing, with the aim of selecting a collection of investment assets that has collectively lower risk than any individual asset. This is possible, intuitively speaking, because different types of assets often change in value in opposite ways. For example, to the extent prices in the stock market move differently from prices in the bond market, a collection of both types of assets can in theory face lower overall risk than either individually.

2. RISK AND EXPECTED RETURN

MVT assumes that investors are risk averse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher expected returns must accept more risk. The exact trade-off will be the same for all investors, but different investors will evaluate the trade-off differently based on individual risk aversion characteristics. The implication is that a rational investor will not invest in a portfolio if a second portfolio exists with a more favorable risk-expected return profile.

3. MATHEMATICAL MODEL

MVT models an asset's return as a normally distributed function (or more generally as an elliptically distributed random variable), defines risk as the standard deviation of return, and models a portfolio as a weighted combination of assets, so that the return of a portfolio is the weighted combination of the assets' returns. By combining different assets whose returns are not perfectly positively correlated, MVT seeks to reduce the total variance of the portfolio return.

The classical MVT model is presented here below:

Under the model:

- Portfolio return is the proportion-weighted combination of the constituent assets' returns.
- Portfolio volatility is a function of the correlations ρ_{ij} of the component assets, for all asset pairs (i, j).

In general:

- Expected return:

$$\mathrm{E}(R_p) = \sum_i w_i \, \mathrm{E}(R_i)$$

where R_p is the return on the portfolio, R_i is the return on asset i and w_i is the weighting of component asset i (that is, the proportion of asset "i" in the portfolio).

- Portfolio return variance:

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij},$$

where ρ_{ij} is the correlation coefficient between the returns on assets i and j. Alternatively the expression can be written as:

$$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij},$$

where $\rho_{ij} = 1$ for $i=j$.

- Portfolio return volatility (standard deviation):

$$\sigma_p = \sqrt{\sigma_p^2}$$

For a **two asset** portfolio:

- Portfolio return: $\mathrm{E}(R_p) = w_A \, \mathrm{E}(R_A) + w_B \, \mathrm{E}(R_B) = w_A \, \mathrm{E}(R_A) + (1 - w_A) \, \mathrm{E}(R_B).$
- Portfolio variance: $\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{AB}$

For a **three asset** portfolio:

- Portfolio return: $w_A \, \mathrm{E}(R_A) + w_B \, \mathrm{E}(R_B) + w_C \, \mathrm{E}(R_C)$

Portfolio variance:

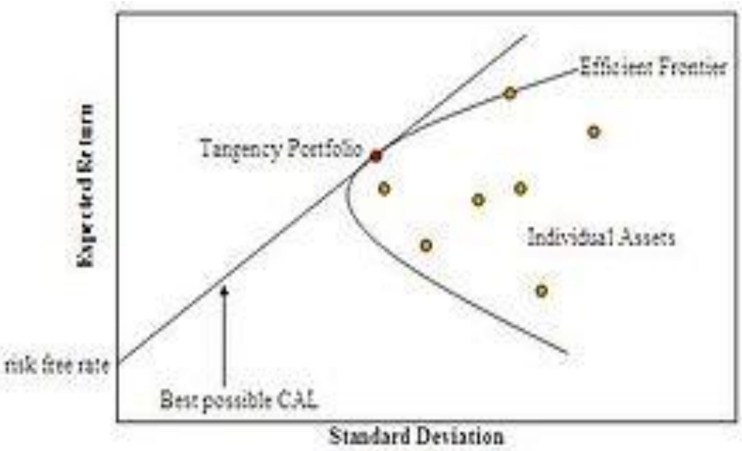
$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{AB} + 2 w_A w_C \sigma_A \sigma_C \rho_{AC} + 2 w_B w_C \sigma_B \sigma_C \rho_{BC}$$

4. DIVERSIFICATION

An investor can reduce portfolio risk simply by holding combinations of instruments that are not perfectly positively correlated (correlation coefficient $-1 \leq \rho_{ij} < 1$). In other words, investors can reduce their exposure to individual asset risk by holding a diversified portfolio of assets. Diversification may allow for the same portfolio expected return with reduced risk.

If all the asset pairs have correlations of 0—they are perfectly uncorrelated—the portfolio's return variance is the sum over all assets of the square of the fraction held in the asset times the asset's return variance (and the portfolio standard deviation is the square root of this sum).

The efficient frontier with no risk-free asset



Efficient Frontier. The hyperbola is sometimes referred to as the 'Markowitz Bullet', and is the efficient frontier if no risk-free asset is available. With a risk-free asset, the straight line is the efficient frontier.

As shown in this graph, every possible combination of the risky assets, without including any holdings of the risk-free asset, can be plotted in risk-expected return space, and the collection of all such possible portfolios defines a region in this space. The left boundary of this region is a hyperbola, and the upper edge of this region is the *efficient frontier* in the absence of a risk-free asset (sometimes called "the Markowitz bullet"). Combinations along this upper edge represent portfolios (including no holdings of the risk-free asset) for which there is lowest risk for a given level of expected return. Equivalently, a portfolio lying on the efficient frontier represents the combination offering the best possible expected return for given risk level.

The risk-free asset and the capital allocation line

The risk-free asset is the (hypothetical) asset that pays a risk-free rate. In practice, short-term government securities (such as US treasury bills) are used as a risk-free asset, because they pay a fixed rate of interest and have exceptionally low default risk. The risk-free asset has zero variance in returns (hence is risk-free); it is also uncorrelated with any other asset (by definition, since its variance is zero). As a result, when it is combined with any other asset or portfolio of assets, the change in return is linearly related to the change in risk as the proportions in the combination vary.

When a risk-free asset is introduced, the half-line shown in the figure is the new efficient frontier. It is tangent to the hyperbola at the pure risky portfolio with the highest Sharpe ratio. This efficient half-line is called the capital allocation line (CAL), and its formula can be shown to be

$$E(R_C) = R_F + \sigma_C \frac{E(R_P) - R_F}{\sigma_P}.$$

In this formula *P* is the sub-portfolio of risky assets at the tangency with the Markowitz bullet, *F* is the risk-free asset, and *C* is a combination of portfolios *P* and *F*.

The introduction of the risk-free asset as a possible component of the portfolio has improved the range of risk-expected return combinations available, because everywhere except at the tangency portfolio the half-line gives a higher expected return than the hyperbola does at every possible risk level.

5. ASSUMPTIONS AND CRITICISMS

- **Investors are interested in the optimization;** selecting a portfolio with highest possible return or lowest possible risk.

In reality, investors have utility functions that may be sensitive to higher moments of the distribution of the returns. For the investors to use the mean-variance optimization, one must suppose that the combination of utility and returns make the optimization of utility problem similar to the mean-variance optimization problem.

- **Asset returns are (jointly) normally or elliptically distributed random variables.**

In fact, it is frequently observed that returns in equity and other markets are not normally distributed. Bouchaud and Chicheportiche (2012) empirically reject the elliptical hypothesis.

Financial economist Nassim Nicholas Taleb has also criticized mean variance portfolio theory:

After the stock market crash (in 1987), they rewarded two theoreticians, Harry Markowitz and William Sharpe, who built beautifully Platonic models on a Gaussian base, contributing to what is called Modern Portfolio Theory. Simply, if you remove their Gaussian assumptions and treat prices as scalable, you are left with hot air. The Nobel Committee could have tested the Sharpe and Markowitz models—they work like quack remedies sold on the Internet—but nobody in Stockholm seems to have thought about it.

- **Correlations between assets are fixed and constant forever.**

Correlations depend on systemic relationships between the underlying assets, and change when these relationships change. Examples include one country declaring war on another, or a general market crash. During times of financial crisis all assets tend to become positively correlated, because they all move (down) together.

- **All investors aim to maximize economic utility (in other words, to make as much money as possible, regardless of any other considerations).**

This is a key assumption of the efficient-market hypothesis, upon which MVT relies.

- **All investors are rational and risk-averse.** This is another assumption of the efficient-market hypothesis.

In reality, as proven by behavioral economics, market participants are not always rational or consistently rational. The assumption does not account for emotional decisions, stale market information, "herd behavior", or investors who may seek risk for the sake of risk. Casino gamblers clearly pay for risk, and it is possible that some stock traders will pay for risk as well.

- **All investors have access to the same information at the same time.**

In fact, real markets contain information asymmetry, insider trading, and those who are simply better informed than others.

- **Investors have an accurate conception of possible returns, i.e., the probability beliefs of investors match the true distribution of returns.**

A different possibility is that investors' expectations are biased, causing market prices to be informationally inefficient.

- **There are no taxes or transaction costs.**

Real financial products are subject both to taxes and transaction costs (such as broker fees), and taking these into account will alter the composition of the optimum portfolio.

- **All investors are price takers, i.e., their actions do not influence prices.**

In reality, sufficiently large sales or purchases of individual assets can shift market prices for that asset and others (via cross elasticity of demand.) An investor may not even be able to assemble the theoretically optimal portfolio if the market moves too much while they are buying the required securities.

- **Any investor can lend and borrow an unlimited amount at the risk free rate of interest.**

In reality, every investor has a credit limit.

- **All securities can be divided into parcels of any size.**

In reality, fractional shares usually cannot be bought or sold, and some assets have minimum orders sizes.

- **Risk/Volatility of an asset is known in advance/is constant**.

In fact, markets often misprice risk (e.g. the US mortgage bubble or the European debt crisis) and volatility changes rapidly.

- **Does not take into account fundamentals affecting asset prices.**

Portfolio managers can invest in assets without analyzing their fundamentals, because the investor purchases assets in proportion to their market weights; there is no relative increase in demand for one asset versus another, and thus no impact on the expected returns of the portfolio.

6. APPLICATIONS

1. By experts to largely project optimal levels of investment in financial assets.

2. By project managers, with some model modifications.

3. In Regional Science: in a series of seminal works, Michael Conroy modeled the labor force in the economy using portfolio-theoretic methods to examine growth and variability in the labor force.

4. Has been used to model the self-concept in social psychology. When the self attributes comprising the self-concept constitute a well-diversified portfolio, then psychological outcomes at the level of the individual such as mood and self-esteem should be more stable than when the self-concept is undiversified. This prediction has been confirmed in studies involving human subjects.

5. Has been recently applied to modelling the uncertainty and correlation between documents in information retrieval. Given a query, the aim is to maximize the overall relevance of a ranked list of documents and at the same time minimize the overall uncertainty of the ranked list.

REFERENCES

[1] Harry M. Markowitz - Autobiography, The Nobel Prizes 1990, Editor Tore Frängsmyr, (Nobel Foundation), Stockholm, 1991

[2] Bhalla, V. K. (2010). *Investment Management*. New Delhi: S. Chand & Co. Ltd. pp. 587–93. ISBN 81-219-1248-2.

[3] Andrei Shleifer: Inefficient Markets: An Introduction to Behavioral Finance. Clarendon Lectures in Economics (2000)

[4] Koponen, Timothy M. 2003. *Commodities in action: measuring embeddedness and imposing values*. The Sociological Review. Volume 50 Issue 4, Pages 543 - 569

[5] Jenson, Michael; Scholes, Myron; Black, Fischer (1972). The Capital Asset Pricing Model: Some Empirical Tests. In Jensen, Michael. *Studies in the Theory of Capital Markets*. Praeger Publishers. Retrieved 2014-03-24.

[6] Edwin J. Elton and Martin J. Gruber, "Modern portfolio theory, 1950 to date", Journal of Banking & Finance 21 (1997) 1743-1759

[7] Markowitz, H.M. (March 1952). "Portfolio Selection". *The Journal of Finance* **7** (1): 77–91. doi:10.2307/2975974. JSTOR 2975974.

[8] Markowitz, H.M. (1959). *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley & Sons. (reprinted by Yale University Press, 1970, ISBN 978-0-300-01372-6; 2nd ed. Basil Blackwell, 1991, ISBN 978-1-55786-108-5)

[9] Merton, Robert. "An analytic derivation of the efficient portfolio frontier," *Journal of Financial and Quantitative Analysis* 7, September 1972, 1851-1872.

[10] Mahdavi Damghani B. (2013).The Non-Misleading Value of Inferred Correlation: An Introduction to the Cointelation Model. *Wilmott Magazine*. doi:10.1002/wilm.10252.

[11] Brodie, De Mol, Daubechies, Giannone and Loris (2009). Sparse and stable Markowitz portfolios. *Proceedings of the National Academy of Sciences* **106** (30). doi:10.1073/pnas.0904287106.

[12] Mandelbrot, B., and Hudson, R. L. (2004). The (Mis)Behaviour of Markets: A Fractal View of Risk, Ruin, and Reward. London: Profile Books.

[13] Chamberlain, G. 1983."A characterization of the distributions that imply mean-variance utility functions", *Journal of Economic Theory* 29, 185-201.

[14] Owen, J.; Rabinovitch, R. (1983). "On the class of elliptical distributions and their applications to the theory of portfolio choice". *Journal of Finance* **38**: 745–752. doi:10.1111/j.1540-6261.1983.tb02499.x.

[15] http://arxiv.org/pdf/1009.1100.pdf Chicheportiche, R., & Bouchaud, J. P. (2012). The joint distribution of stock returns is not elliptical. International Journal of Theoretical and Applied Finance, 15(03).

[16] 'Overconfidence, Arbitrage, and Equilibrium Asset Pricing,' Kent D. Daniel, David Hirshleifer and Avanidhar Subrahmanyam, Journal of Finance, 56(3) (June, 2001), pp. 921-965

[17] Taleb, Nassim Nicholas (2007), *The Black Swan: The Impact of the Highly Improbable, Random House, ISBN 978-1-4000- 6351-2.*

[18] Hubbard, Douglas (2007). *How to Measure Anything: Finding the Value of Intangibles in Business*. Hoboken, NJ: John Wiley & Sons. ISBN 978-0-470-11012-6.

[19] Sabbadini, Tony (2010). Manufacturing Portfolio Theory. International Institute for Advanced Studies in Systems Research and Cybernetics.

[20] Chandra, Siddharth (2003). "Regional Economy Size and the Growth-Instability Frontier: Evidence from Europe". *Journal of Regional Science* **43** (1): 95–122. doi:10.1111/1467-9787.00291.

[21] Chandra, Siddharth; Shadel, William G. (2007). "Crossing disciplinary boundaries: Applying financial portfolio theory to model the organization of the self-concept". *Journal of Research in Personality* **41** (2): 346–373. doi:10.1016/j.jrp.2006.04.007.

[22] Portfolio Theory of Information Retrieval July 11th, 2009 (2009-07-11). "Portfolio Theory of Information Retrieval | Dr. Jun Wang's Home Page". Web4.cs.ucl.ac.uk. Retrieved 2012-09-05.

[23] Lintner, John (1965). "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets". *The Review of Economics and Statistics* (The MIT Press) 47 (1): 13–39. doi:10.2307/1924119. JSTOR 1924119.

[24] Sharpe, William F. (1964). "Capital asset prices: A theory of market equilibrium under conditions of risk". *Journal of Finance* **19** (3): 425–442. doi:10.2307/2977928. JSTOR 2977928.

[25] Tobin, James (1958). "Liquidity preference as behavior towards risk". *The Review of Economic Studies* 25 (2): 65–86. doi:10.2307/2296205. JSTOR 2296205.